Introduction to Chemical Engineering

Chapter 05

Material Balances
(*How Much Base Do We Need?*)

5.1 Conservation of total mass

- One important principle in dealing with material balances is that **total mass is conserved**.
- All mass entering a system will either leave that same system or will accumulate in the system.

Rate that mass enters the system = Rate that mass leaves the system + Rate that mass accumulates in the system (5.1)

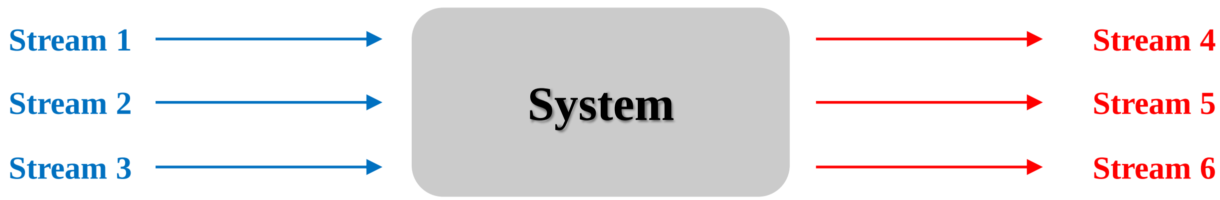
*Unlike total mass, total moles are not always conserved.

- At steady-state process,

Rate that mass enters the system = Rate that mass leaves the system + ~~Rate that mass accumulates in the system~~ → 0 (5.2)

5.1 Conservation of total mass

A schematic diagram of a general system with input and output streams



$$\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4 + \dot{m}_5 + \dot{m}_6$$

Steady-State Total Mass Balance

$$\sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_{in} = \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_{out} \quad (5.3) \quad \xrightarrow{\dot{m} = \rho\dot{V} \ (5.4a)} \quad \sum_{\substack{\text{input}\\\text{streams}}} \left(\rho\dot{V}\right)_{in} = \sum_{\substack{\text{output}\\\text{streams}}} \left(\rho\dot{V}\right)_{out}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_{in} = \sum_{\substack{\text{output}\\\text{streams}}} \left(\rho\dot{V}\right)_{out}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} \left(\rho\dot{V}\right)_{in} = \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_{out}$$

5.1 Conservation of total mass

Example 5.2

Three different streams deliver contaminated oil to a waste oil tank, and the mass or volumetric flow rates of those streams are given below:

Input stream	Volumetric flow rate (*gal/hr*)	Density (lb_m/ft^3)	Mass flow rate (lb_m/hr)
1	27.4	53.7	
2			243.9
3	19.4	46.0	

If the oil leaving the tank has a density of 50.8 lb_m/ft^3, at what volumetric flow rate must the oil be withdrawn to maintain a constant scale reading?

5.1 Conservation of total mass

Steps for analyzing material balance problems

1. **Draw a diagram** if one is not already available.
2. **Write all *known* quantities** (flow rates, densities, etc.) in the appropriate locations on the diagram. If symbols are used to designate known quantities, include those symbols on the diagram.
3. **Identify and assign symbols to all *unknown* quantities** and write them in the appropriate locations on the diagram.
4. **Select a basis if needed**: if no flow rates are known, assume a convenient value for one of the flow rates as a basis of calculation (e.g., 100 *lbmol/s*, 100 *kg/hr*, etc.).
5. **Determine the appropriate set of equations** needed to solve for the unknown quantities. In order for the problem to be solved, the number of equations must equal the number of unknowns. The steps below can be used to obtain the desired set of equations.
    a. Construct the material balance equation(s):
        1. Start with the general equation
        2. Discard terms that equal zero in your specific problem
        3. Replace remaining terms with more convenient forms
    b. Construct equations to express other known relationships between variables
6. **Solve algebraically and then numerically**: solve algebraically for the desired parameters and then determine their values.

5.1 Conservation of total mass

Example 5.3

Your company uses a process to concentrate orange juice by freeze-drying. The input to the process is orange juice that has a density of 1.01 *g/cm³*. Two streams are output from the process. The first output stream is the orange juice concentrate. The second is an ice slurry that has a density of 0.93 *g/cm³*. The orange juice is concentrated such that the volume of the concentrate is ¼ that of the incoming juice. Also, from experience it is known that we want to remove ice slurry at the following volumetric rate (where J = juice and I = ice):

$$\dot{V}_I = 0.7\frac{\rho_J \dot{V}_J}{\rho_I}$$

What is the density of the concentrate?

5.2 Material balances for multiple species

Mass balance for species A

Rate that A enters the system + Rate that A is formed in the system = Rate that A leaves the system + Rate that A is consumed in the system + Rate that A accumulates in the system (5.5)

At steady-state process,

Rate that A enters the system + Rate that A is formed in the system = Rate that A leaves the system + Rate that A is consumed in the system + ~~Rate that A accumulates in the system~~ → 0 (5.6)

Steady-State Mass Balance for Species A

$$\sum_{\text{input streams}} \dot{m}_{A,in} + R_{formation,A} = \sum_{\text{output streams}} \dot{m}_{A,out} + R_{consumption,A} \qquad (5.7)$$

$R_{formation,A}$ = *rate that species A is formed, in units of mass / time*

$R_{consumption,A}$ = *rate that species A is consumed, in units of mass / time*

5.2 Material balances for multiple species

Example 5.4

Natural gas, which is essentially pure methane, undergoes steady-state combustion by injecting it into a small burner into which air is also injected. The methane flow rates in the steady input and output streams are

- Natural gas input stream: Methane mass flow rate = 4.61 *g/s*
- Air input stream: Contains no methane
- Output (flue gas) stream: Flow rate of unburned methane = 0.09 *g/s*

At what rate is the methane being burned (consumed)?

5.2 Material balances for multiple species

Other expressions of mass and molar flow rates

$$\dot{m}_A = x_A\dot{m} = MW_A\dot{n}_A = MW_A y_A \dot{n} = MW_A c_A \dot{V} \quad (5.8a)$$

$$\dot{n}_A = \frac{\dot{m}_A}{MW_A} = \frac{x_A\dot{m}}{MW_A} = y_A\dot{n} = c_A\dot{V} \quad (5.8b)$$

Many forms of the material balance for species A

$$\sum_{\substack{\text{input}\\\text{streams}}} \dot{m}_{A,in} + R_{formation,A} = \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_{A,out} + R_{consumption,A}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} x_A\dot{m}_{in} + R_{formation,A} = \sum_{\substack{\text{output}\\\text{streams}}} MW_A\dot{n}_{A,out} + R_{consumption,A}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} MW_A\dot{n}_{in} + R_{formation,A} = \sum_{\substack{\text{output}\\\text{streams}}} MW_A\left(c_A\dot{V}\right)_{out} + R_{consumption,A}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} MW_A\left(y_A\dot{n}\right)_{in} + R_{formation,A} = \sum_{\substack{\text{output}\\\text{streams}}} \dot{m}_{A,out} + R_{consumption,A}$$

$$\sum_{\substack{\text{input}\\\text{streams}}} MW_A\left(c_A\dot{V}\right)_{in} + R_{formation,A} = \sum_{\substack{\text{output}\\\text{streams}}} x_A\dot{m}_{out} + R_{consumption,A}$$

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5.2 Material balances for multiple species

5.2.1 Material balances with formation or consumption where chemical reaction stoichiometry is not given

Example 5.5

Penicillin is produced in reactors containing the bacteria of the species *Penicililium chrysogenum*. One such method involves continuous (steady-state) production in a continuously stirred tank reactor, where optimum penicillin production has been reported when both of the following are true:

1. the penicillin concentration inside the reactor is 0.002 *gmol/L*

2. the inlet volumetric flow rate is 0.25 *L/hr* for a 10 *L* reactor

A nutrient stream (containing no penicillin) is fed to a 10 *L* reactor containing the *Penicililium chrysogenum* bacteria. A product stream containing penicillin leaves the reactor (the bacteria stay in the reactor, and the penicillin concentration in the product stream is the same as inside the reactor). The densities of the nutrient and product streams can be assumed to be equal.

What is the production rate of penicillin under these conditions?

Note: The molecular weight of this penicillin can be taken as 334.4.

5.2 Material balances for multiple species

5.2.2 Material balances with no formation/consumption

Example 5.6

Benzene and toluene (two similar compounds) are partially separated using a distillation column. The feed (input) stream of 100 *kg/hr* contains benzene at a mass fraction of 0.40, with the balance being toluene. In the overhead output stream, the benzene flow rate is 36 *kg/hr*, and in the bottoms output stream, the toluene flow rate is 54 *kg/hr*. What are the toluene flow rate in the overhead output stream, and the benzene flow rate in the bottoms output stream?

5.2 Material balances for multiple species

5.2.3 Material balances with formation/consumption where chemical reaction stoichiometry is given

- Symbols of formation/consumption rate

$$r_{formation,A} = \text{rate that species } A \text{ is formed, in units of moles / time}, \quad R_{formation,A} = MW_A r_{formation,A} \tag{5.9a}$$

$$r_{consumption,A} = \text{rate that species } A \text{ is consumed, in units of moles / time}, \quad R_{consumption,A} = MW_A r_{consumption,A} \tag{5.9b}$$

- Steady-State Mole Balance for Species A

$$\sum_{\substack{input\\streams}} \dot{n}_{A,in} + r_{formation,A} = \sum_{\substack{output\\streams}} \dot{n}_{A,out} + r_{consumption,A} \tag{5.10}$$

Other forms…

$$\sum_{\substack{input\\streams}} (y_A\dot{n})_{in} + r_{formation,A} = \sum_{\substack{output\\streams}} \dot{n}_{A,out} + r_{consumption,A}$$

$$\sum_{\substack{input\\streams}} (c_A\dot{V})_{in} + r_{formation,A} = \sum_{\substack{output\\streams}} (y_A\dot{n})_{out} + r_{consumption,A}$$

5.2 Material balances for multiple species

5.2.3 Material balances with formation/consumption where chemical reaction stoichiometry is given

- Chemical reaction with stoichiometric coefficients

$$\nu_A A + \nu_B B \rightarrow \nu_C C + \nu_D D$$

$$\frac{r_{consumption,B}}{r_{consumption,A}} = \frac{\nu_B}{\nu_A} \qquad \frac{r_{formation,C}}{r_{consumption,A}} = \frac{\nu_C}{\nu_A} \qquad \frac{r_{formation,D}}{r_{consumption,A}} = \frac{\nu_D}{\nu_A}$$

- Consumption rate of A with the fractional conversion of A

$$r_{consumption,A} = X_A \sum_{\substack{All\ Input \\ Streams}} \dot{n}_{A,in}$$

For example, an 85% conversion of A means that 85% of the A that enters the reactor is consumed in the reaction.

5.2.3 Material balances with formation/consumption where chemical reaction stoichiometry is given

Example 5.7

Ethylene is used to make many important products, including the common polymer polyethylene (used to make plastic bags and many other products). As a design engineer, you are designing a process to convert some excess butene (C_4H_8) to ethylene (C_2H_4) using the reaction

$$C_4H_8 \rightarrow 2C_2H_4$$

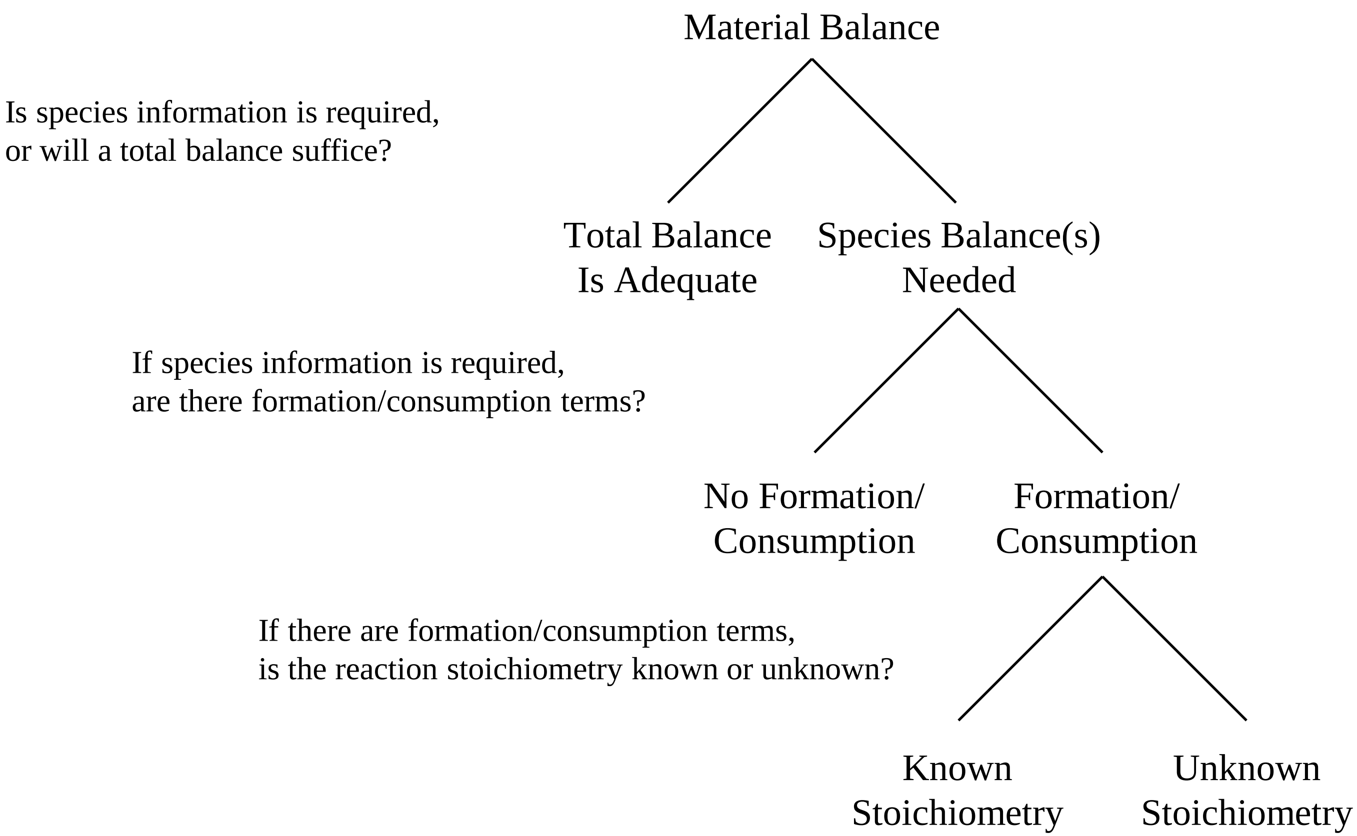
Your process will include reaction and separation, and two streams will exit the process: an ethylene-rich stream and a butene-rich stream. The following are requirements for the process:

Feed (input) stream:	Flow rate: 865 lb_m/hr Composition: Pure butene
Ethylene-rich output stream:	Flow rate: 25.5 *lbmol/hr* Composition: 92 mole% ethylene (the rest is butene)
Butene-rich output stream:	Flow rate: 240 ft^3/hr Composition: ethylene and butene

Of the incoming butene, 84% is converted to ethylene. What will be the concentrations of ethylene and butene (in units of $lbmol/ft^3$) in the butene-rich outlet stream?

5.3 Material balances: summary

Decision-tree diagram



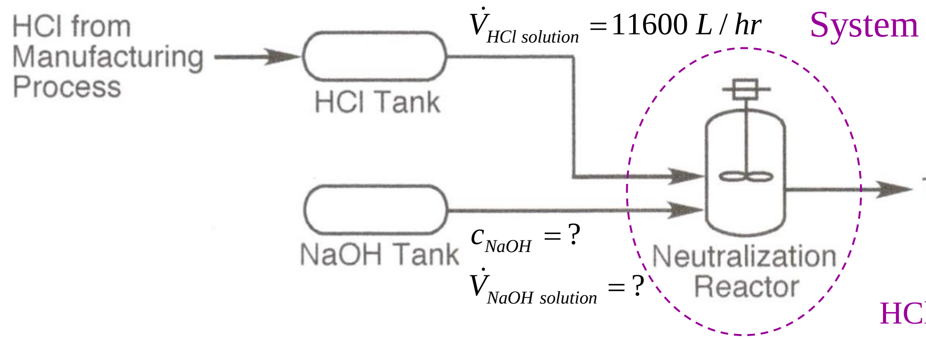
5.3 Material balances: summary

5.3.1 The acid-neutralization problem

$c_{HCl} = 0.014\ M$

$\dot{V}_{HCl\ solution} = 11600\ L/hr$

System



$c_{NaOH} = ?$

$\dot{V}_{NaOH\ solution} = ?$

$c_{HCl} = c_{NaOH} = 0\ M$

$\frac{r_{consumption,HCl}}{r_{consumption,NaOH}} = \frac{1}{1} = 1$

$HCl + NaOH \rightarrow H_2O + NaCl$

Steady-State Mole Balance for HCl

$$\sum_{\substack{input\\streams}} \left(c_{HCl}\dot{V}_{HCl\ solution}\right)_{in} + r_{formation,HCl} = \sum_{\substack{output\\streams}} \left(c_{HCl}\dot{V}_{HCl\ solution}\right)_{out} + r_{consumption,HCl}$$

$r_{formation,HCl} = 0$, $\sum_{output\ streams}\left(c_{HCl}\dot{V}_{HCl\ solution}\right)_{out} = 0$

$$r_{consumption,HCl} = r_{consumption,NaOH}$$

Steady-State Mole Balance for NaOH

$$\sum_{\substack{input\\streams}} \left(c_{NaOH}\dot{V}_{NaOH\ solution}\right)_{in} + r_{formation,NaOH} = \sum_{\substack{output\\streams}} \left(c_{NaOH}\dot{V}_{NaOH\ solution}\right)_{out} + r_{consumption,NaOH}$$

$r_{formation,NaOH} = 0$, $\sum_{output\ streams}\left(c_{NaOH}\dot{V}_{NaOH\ solution}\right)_{out} = 0$

$$\rightarrow c_{NaOH}\dot{V}_{NaOH\ solution} = c_{HCl}\dot{V}_{HCl\ solution}$$

$$\dot{V}_{NaOH\ solution} = \frac{c_{HCl}\dot{V}_{HCl\ solution}}{c_{NaOH}}$$

Homework problems

Homework problem 1.

Feed water is fed to a large steady-state boiler. Most of the water leaves the boiler as high-pressure steam, with a smaller amount of hot residual water discharged to waste. The water densities and flow rates and the steam density are as follows:

Stream	Density (kg/m^3)	Volumetric Flow Rate (m^3/min)
Feed water	1000	28.0
Hot residual water	960	6.5
High-pressure steam	3.7	

What is the volumetric flow rate of the steam?

Homework problems

Homework problem 5.

An "absorber" is a device used to bring a gas containing toxic chemicals into contact with a liquid that absorbs some of those toxic chemicals from the gas. Thus, a gas stream and liquid stream leave the absorber.
In an absorber you are designing, the entering gas stream flows at 340 ft^3/min, while the gas stream leaving the absorber flows at 270 ft^3/min. Meanwhile, the liquid stream leaving the absorber must be treated, so its flow rate is specified to be 77 lb_m/min.
What mass flow rate is needed for the entering liquid stream (in units of lb_m/min)?
The densities of the entering and exiting gas streams are equal to 0.087 lb_m/ft^3.

Homework problems

Homework problem 10.

Gas in a tank contains a poison at a concentration c_T, but the gas is leaking into the surrounding room at a rate of T cm^3/min. Meanwhile, an air conditioner brings fresh air into the room at a rate of A cm^3/min. The air in the room is well mixed and leaves through an open window (with the same concentration as in the room). The densities of all the gases are the same. Initially, the concentration of poison in the room (c_R) will rise, but it eventually will reach a steady value. In terms of the given symbols, what is that steady concentration?

Homework problems

Homework problem 13.

For the combustion of methane presented in Example 5.4, the chemical reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Suppose that methane flows into a burner at 30 *gmol/s*, while oxygen flows into the same burner at 75 *gmol/s*. If all the methane is burned and a single output stream leaves the burner, what is the mole fraction of CO_2 in that output stream?

Hint 1: Does the fact that all the methane is burned mean that all the oxygen is burned also?

Hint 2: Find the molar flow rate of each component gas in the outlet gas ("flue gas").

Homework problems

Homework problem 16.

Fuel cells have been proposed for use in cars and for power generation as part of a hydrogen economy. They offer the advantages of higher efficiency (not limited by heat cycle efficiencies) and cleaner fuel with no significant pollutants. The net reaction is as follows:

$$2H_2 + O_2 \rightarrow 2H_2O$$

where O_2 comes from the air (which, for this problem, can be assumed to be 21 mole% O_2 and 79 mole% N_2). Hydrogen (H_2) flows into a fuel cell implemented in a prototype vehicle at a rate of 27 *gmol/min*. Air (consisting of oxygen and nitrogen enters the fuel cell in a separate stream. The amount of oxygen entering the fuel cell is 50% more than that needed to react stoichiometrically with all of the entering hydrogen. The conversion of hydrogen in the fuel cell is 85%. Assume that only a single stream exits the fuel cell. What is the flow rate of each of the species leaving the fuel cell? Note that the flow rates given correspond to a fuel cell rated at approximately 50 *kW* using 2004 technology.

Homework problems

Homework problem 18.

Ammonia is used for fertilizer production and has been critical to successful agriculture. A steady-state chemical process is used to convert nitrogen (N_2) and hydrogen (H_2) to ammonia (NH_3) by the reaction

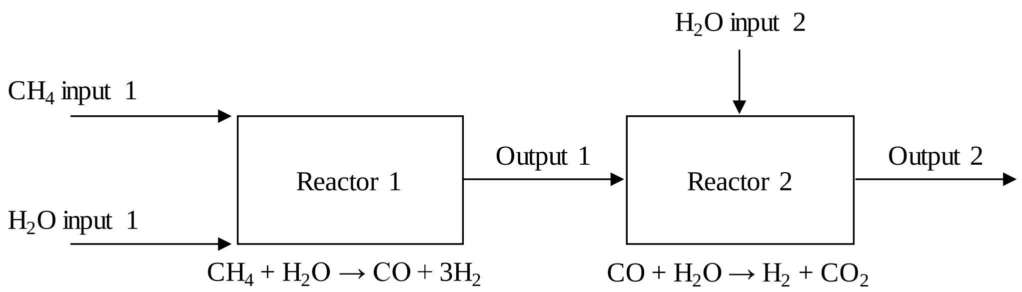
$$N_2 + 3H_2 \rightarrow 2NH_3$$

Stream 1, containing 95 mole% nitrogen and 5 mole% hydrogen, enters the process at a rate of 400 *lbmol/hr*, and stream 2, containing pure hydrogen (density = 0.08 lb_m/ft^3), enters the process at a volumetric flow rate of 31000 ft^3/hr. A single stream leaves the process. If all of the nitrogen is consumed in the reaction, what is the molar flow rate of hydrogen in the exiting stream?

Homework problems

Problem 05 (Mid-term Exam 2018)

(30 point) In order to produce high quality hydrogen from methane, two steady-state reactors are serially connected as shown in below.



For the Reactor 1, the methane steam reforming reaction occurs ($CH_4 + H_2O \rightarrow CO + 3H_2$). The methane input flows into the Reactor 1 at a rate of 64.2 g/min. Water input enters the Reactor 1 in a separate stream. The amount of water entering the Reactor 1 is 25% more than that needed to react stoichiometrically with all of the entering methane. The conversion of methane in Reactor 1 is 75%.

For the Reactor 2, the water gas shift reaction occurs ($CO + H_2O \rightarrow H_2 + CO_2$). The output from Reactor 1 flows into the Reactor 2. Another water input enters the Reactor 2 in a separate stream, in order to achieve the carbon monoxide conversion of 100%.

Assume that there is no side reaction in both of the Reactor 1 and 2.

(a) What is mole fraction of CO in the output from Reactor 1?
(b) What is the minimum molar flow rate of H_2O input 2 entering the Reactor 2? The answer should be given with unit of gmol/min.
(c) What is the mole fraction of H_2 in the output from Reactor 2?