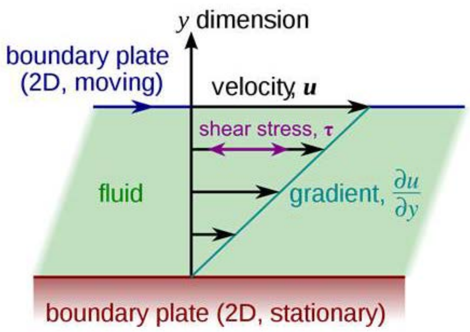
Introduction to Chemical Engineering

Chapter 07

Fluid Flow
(*Bringing the Base to the Acid*)

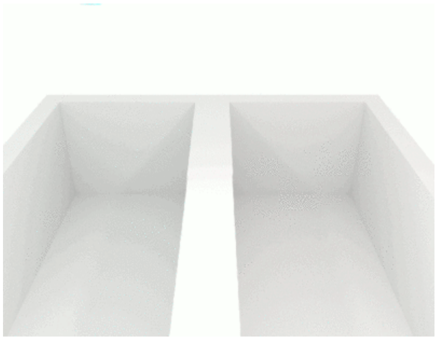
7.1 What is a fluid?

- **A fluid is a substance that continually deforms (flows) under an applied shear stress.**



$$Newtonian\ \ fluid: \tau = \mu \frac{\partial u}{\partial y}$$

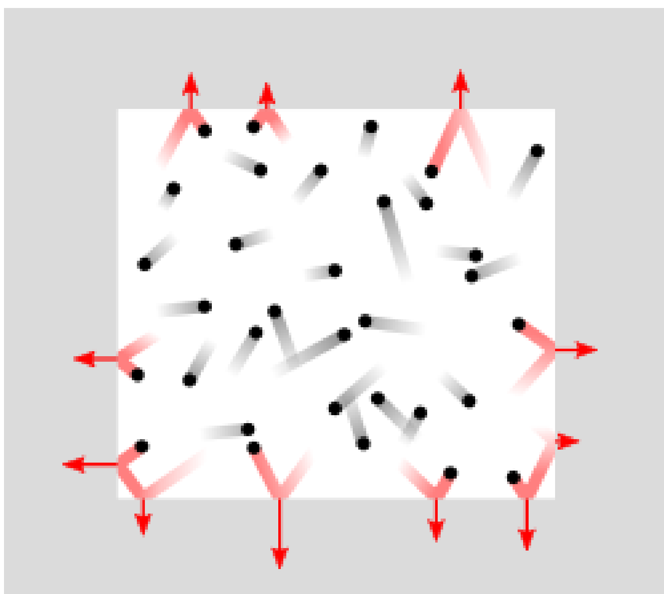
$$Incompressible\ \ fluid: \rho = constant$$

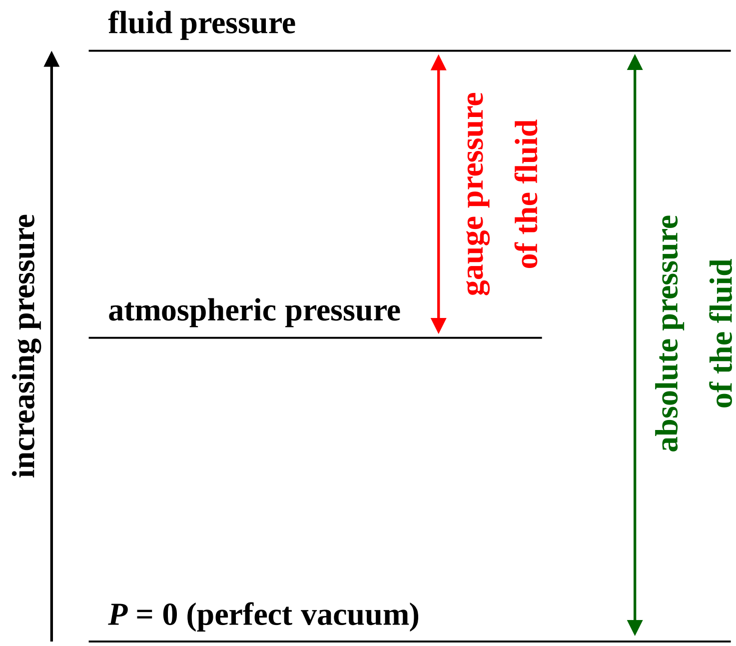


- **Fluids are a subset of the phases of matter and include liquids, gases, plasmas and to some extent, plastic solids.**

7.2 The concept of pressure

- **Pressure is the force applied perpendicular to the surface of an object per unit area over which that force is distributed.**





$$Gauge\ pressure = Absolute\ pressure - Atmospheric\ pressure \tag{7.1}$$

7.2 The concept of pressure

- **Some common units of pressure**

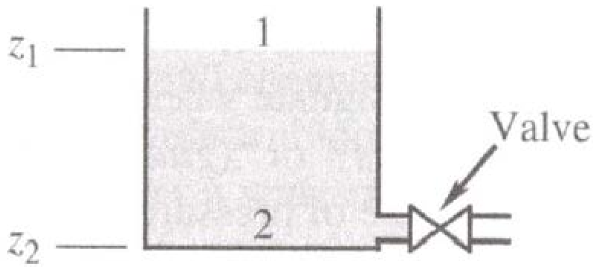
Unit name	Abbreviation	Quantity in 1 atmosphere
pounds (lb_f) per square inch	*psi*	14.7
Pascals	*Pa*	101300
atmospheres	*atm*	1.0
millimeters of Mercury	*mm Hg*	760

Example 7.1

A man pumps up his automobile tire until the tire gauge reads 34.0 *psi*. If the atmospheric pressure in his community is 14.2 *psia*, what is the absolute pressure of the air in the tire?

7.3 Non-flowing fluids

- **Pressure in stagnant (non-flowing) liquids**



$$P_2 - P_1 = \rho g(z_1 - z_2) \tag{7.2}$$

Example 7.2

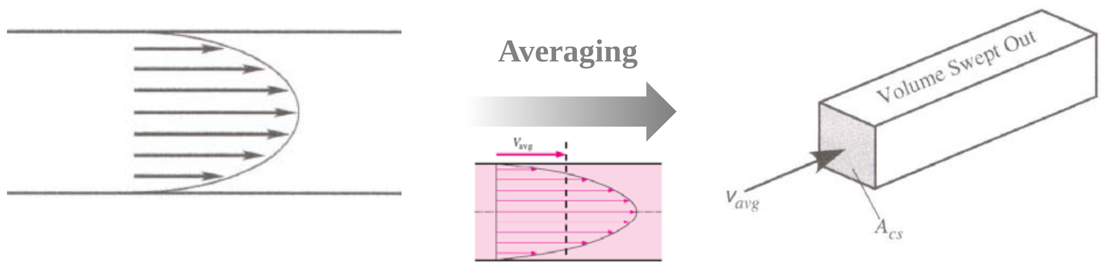
For the tank depicted in above, if the height of the NaOH solution is 8 *ft* (2.44 *m*), what is the pressure at the bottom of the tank? Assume that the density of the NaOH solution is the same as that of water (62.4 lb_m/ft^3 = 1000 kg/m^3). Perform the calculation both in American units and metric units.

Example 7.3

By what fraction will the pressure at the bottom of a tank decrease if the tank is drained to reduce the height of its liquid contents by 35%?

7.4 Principles of fluid flow

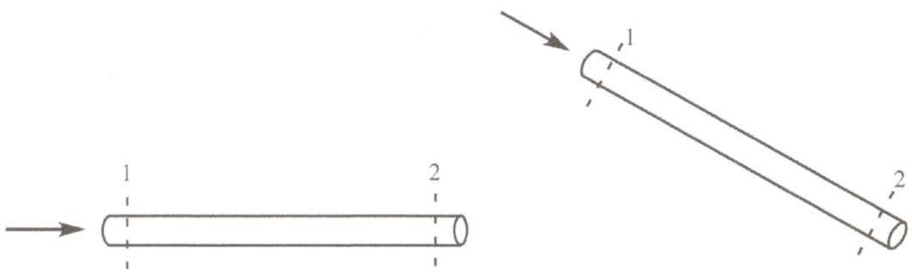
➢ **Velocity profiles of a fluid in a circular pipe**



$$\dot{V} = v_{avg} A_{cs} \qquad (7.3)$$

$$\rho\dot{V} = \dot{m} = \rho v_{avg} A_{cs} \qquad (7.4)$$

➢ **Mass balances in horizontal and downward-inclined sections of pipes**



$$\dot{m}_1 = \dot{m}_2 \quad or \quad \rho_1 v_{avg,1} A_{cs,1} = \rho_2 v_{avg,2} A_{cs,2} \xrightarrow[\text{Same cross-sectional area}]{\text{Incompressible fluid}} v_{avg,1} = v_{avg,2}$$

7.4 Principles of fluid flow

➢ **Mechanical energy equation for steady-state incompressible flow**

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{avg}^{2}+gz\right)_{2}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{avg}^{2}+gz\right)_{1}=w_{s}-w_{f} \qquad \text{(units: } \textit{energy per mass of fluid}\text{)} \qquad (7.8a)$$

Kinetic energy per mass
: translational motion

Potential energy per mass
: elevation

Energy associated with pressure
: storing mechanical energy

Work
: shaft work

Friction
: frictional effects

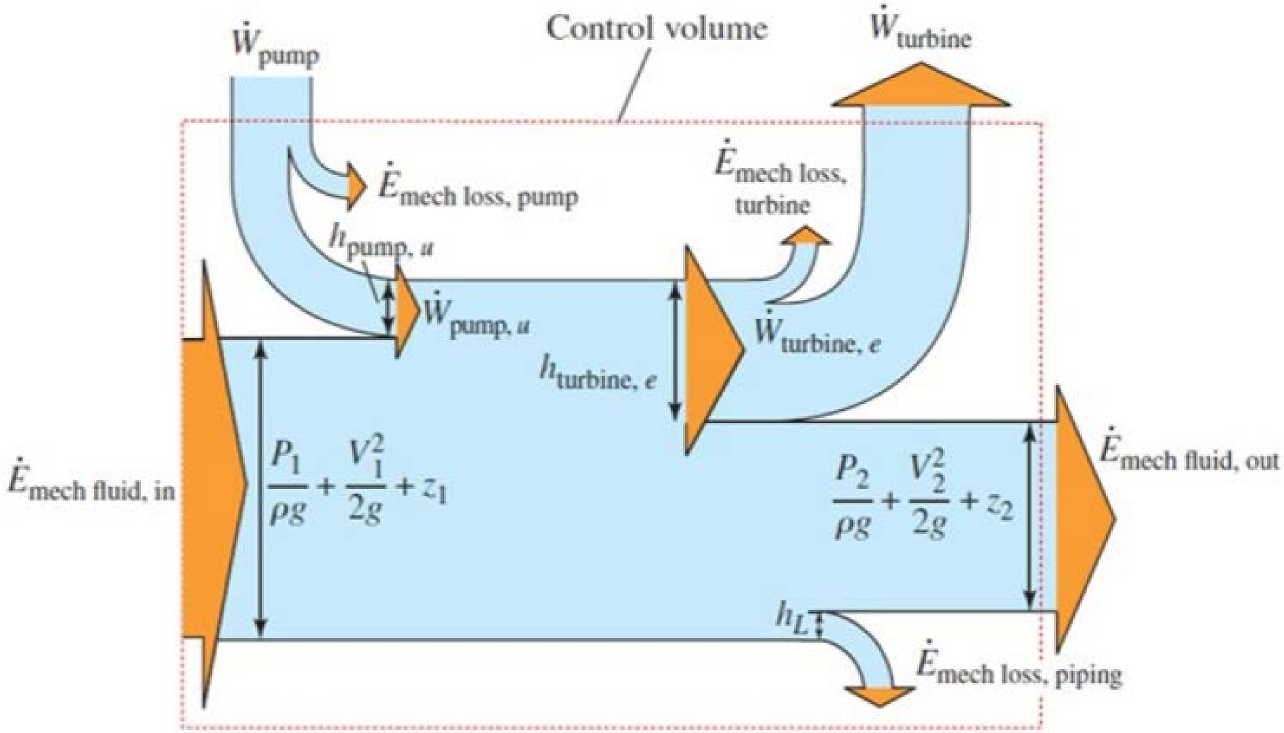
***The correction factor (α) is 2.0 for fully developed laminar pipe flow, and it ranges between 1.04 and 1.11 for fully developed turbulent flow in a circular pipe.**

Other form:

$$\frac{P_2-P_1}{\rho}+\frac{1}{2}\left(\alpha_2 v_{2,avg}^{2}-\alpha_1 v_{1,avg}^{2}\right)+g\left(z_2-z_1\right)=w_s-w_f \qquad (7.8b)$$

7.4 Principles of fluid flow

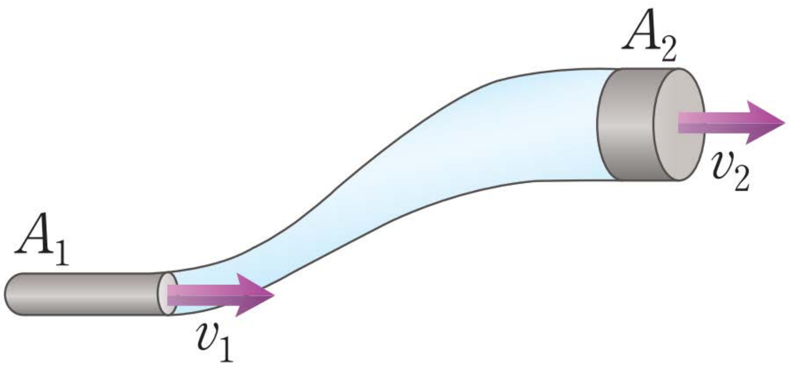
- **Mechanical energy equation for steady-state incompressible flow**



$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\left(\alpha_2 v_{2,avg}^2 - \alpha_1 v_{1,avg}^2\right) + g\left(z_2 - z_1\right) = w_s - w_f$$

7.4 Principles of fluid flow

7.4.1 Special case: no friction (w_f = 0) or shaft work (w_s = 0)



- **Bernoulli equation**

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{avg}^{2}+gz\right)_2-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{avg}^{2}+gz\right)_1=0 \qquad (7.9a)$$

$$\frac{P_2-P_1}{\rho}+\frac{1}{2}\left(\alpha_2 v_{2,avg}^{2}-\alpha_1 v_{1,avg}^{2}\right)+g\left(z_2-z_1\right)=0 \qquad (7.9b)$$

$$\dot{m}=\rho_1 A_{cs,1} v_{avg,1}=\rho_2 A_{cs,2} v_{avg,2} \qquad (7.10)$$

7.4 Principles of fluid flow

Example 7.4

Octane (density = 44.1 lb_m/ft^3) flows downward in a vertical tube at a velocity of 9.9 *in/s*. At a certain location, the pressure is known to be 1.60 *psig*. Just below that point, the diameter of the tube is reduced by one-half, and the velocity becomes 39.6 *in/s*. What is the pressure in the reduced section at a distance 3 *in* below the point where the pressure is known? All values of α can be assumed to equal 1.0, and friction can be ignored.

Example 7.5

Octane (density = 44.1 lb_m/ft^3) flows downward in a tube (inside diameter = 0.7 *in*) at a mass flow rate of 5.83 lb_m/min. At a certain location, the pressure is known to be 1.60 *psig*. Just below that point, the diameter of the tube is reduced by one-half (inside diameter = 0.35 *in*). What is the pressure in the reduced section at a distance 3 *in* below the original location where the pressure is known? Again, all values of α can be assumed to equal 1.0, and friction can be ignored.

7.4 Principles of fluid flow

7.4.2 Generalized solution procedure

1. **Select the two reference locations 1 and 2.**
2. **Eliminate terms that are known to equal 0.**
3. **Determine the other velocities for expansions and contractions.**
4. **Solve for unknown value.**

7.4 Principles of fluid flow

7.4.3 The effects of fluid friction

Example 7.6

How are the inlet and outlet pressures related in a horizontal pipe of constant diameter?

First, select the entrance and exit of the pipe as location 1 and 2, respectively.

For a horizontal pipe, $z_1 = z_2$ (i.e., the potential energy does not change).
For constant diameter, $v_{1,avg} = v_{2,avg}$ (i.e., the kinetic energy does not change).
With no moving parts, $w_s = 0$.

Thus, the mechanical energy equation becomes

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\cancelto{0}{\left(\alpha_2 v_{2,avg}^2 - \alpha_1 v_{1,avg}^2\right)} + g\cancelto{0}{\left(z_2 - z_1\right)} = \cancelto{0}{w_s} - w_f \quad \Longrightarrow \quad \frac{P_2 - P_1}{\rho} = -w_f \quad or \quad \frac{P_1 - P_2}{\rho} = w_f$$

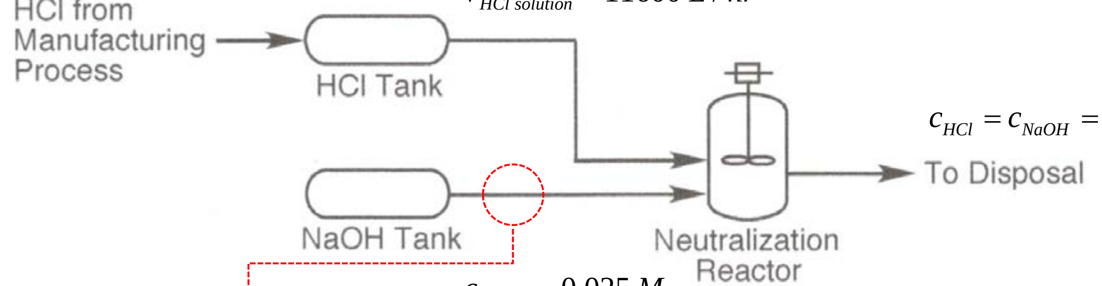
Rearranging: $P_1 = P_2 + \rho w_f$

In steady flow through a constant-diameter horizontal pipe, fluid flows through the pipe because the pressure is higher at the inlet end than at the outlet end, and, for constant fluid density, that difference in pressure divided by the density is equal to the friction produced by the flow.

7.4 Principles of fluid flow

7.4.4 Pumps are used to provide a consistent flow

$$c_{HCl} = 0.014\ M$$
$$\dot{V}_{HCl\ solution} = 11600\ L/hr$$



$$c_{HCl} = c_{NaOH} = 0\ M$$

$$c_{NaOH} = 0.025\ M$$
$$\dot{V}_{NaOH\ solution} = 6496\ L/hr$$

We need something here
to keep volumetric flow rate at constant.

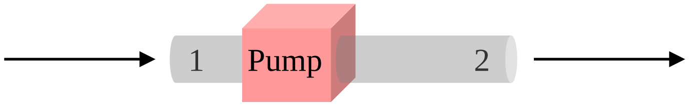
7.5 Pumps and turbines

7.5.1 Pumps

Example 7.8

What is the relationship between the inlet and outlet pressures in a horizontal pipe of constant diameter when a pump is providing flow?

Selecting the entrance and exit of the pipe as location 1 and 2, respectively.



For a horizontal pipe, $z_1 = z_2$.
For constant diameter, $v_{1,avg} = v_{2,avg}$.

Now, the mechanical energy equation becomes

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\underbrace{\cancel{\left(\alpha_2 v_{2,avg}^2 - \alpha_1 v_{1,avg}^2\right)}}_{0} + g\underbrace{\cancel{\left(z_2 - z_1\right)}}_{0} = w_s - w_f \quad \Longrightarrow \quad \frac{P_2 - P_1}{\rho} = w_s - w_f$$

Rearranging: $P_2 = P_1 + \rho w_s - \rho w_f$

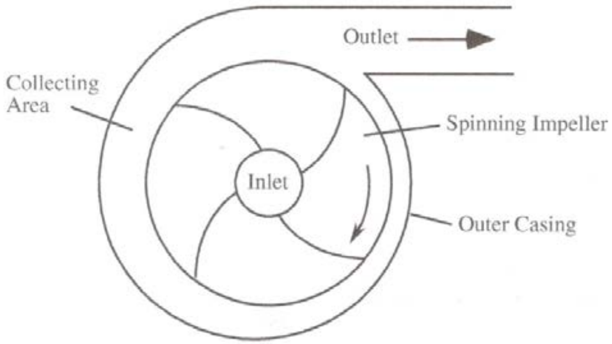
Since the shaft work being done on the liquid is coming only from the work being delivered by the pump, $w_s = w_{pump}$,

$$P_2 = P_1 + \rho w_{pump} - \rho w_f$$

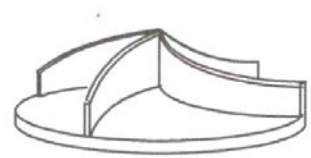
7.5 Pumps and turbines

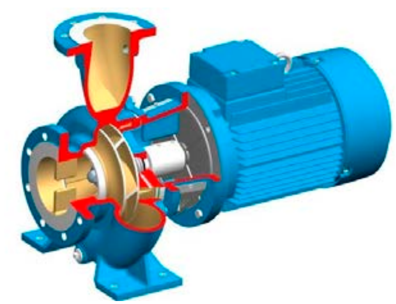
7.5.1 Pumps

- **Centrifugal pumps**

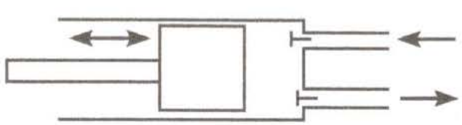


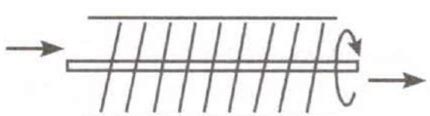
Inlet View of a Centrifugal Pump

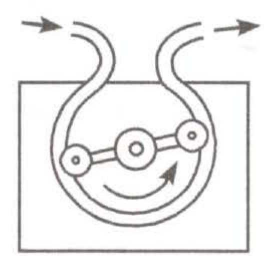

Side View of Impeller



- **Positive-displacement pumps**


Piston-Cylinder


Screw-Type

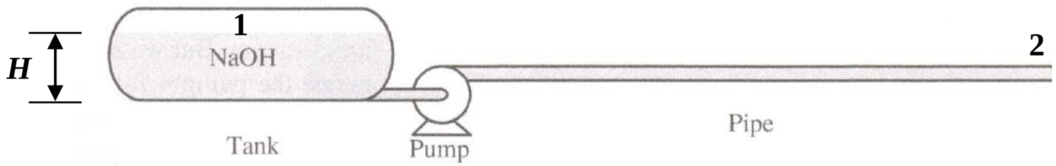

Peristaltic Pump

7.5 Pumps and turbines

7.5.1 Pumps

Example 7.9

If the height of the NaOH solution in the tank is H, how much work must the pump do to deliver the NaOH solution just to the reactor shown in Figure 7.10 (i.e., as though the exit end of the pipe was open to the atmosphere)?



For the open tank and pipe outlet, $P_1 = P_2 = P_{atm} = 0$ (gauge).
For a large tank, $v_{1,avg} \approx 0$.
Thus, the mechanical energy equation becomes

$$\underbrace{\cancel{\frac{P_2 - P_1}{\rho}}}_{0} + \frac{1}{2}\left(\alpha_2 v_{2,avg}^2 - \underbrace{\cancel{\alpha_1 v_{1,avg}^2}}_{0}\right) + g\left(z_2 - z_1\right) = w_s - w_f \quad \Longrightarrow \quad gz_2 + \frac{1}{2}\alpha_2 v_{2,avg}^2 - gz_1 = w_s - w_f$$

Recalling the definition of w_{pump} and rearranging,

$$w_{pump} = w_s = w_f + \frac{1}{2}\alpha_2 v_{2,avg}^2 - g\left(z_1 - z_2\right) \quad or \quad w_{pump} = w_f + \frac{1}{2}\alpha_2 v_{2,avg}^2 - gH$$

7.5 Pumps and turbines

7.5.1 Pumps

➢ **Power of pump**

$$Power = \frac{work}{time} = \left(\frac{mass}{time}\right)\left(\frac{work}{mass}\right) = \dot{m}w_s = \rho\dot{V}w_s \qquad (7.11)$$

➢ **Efficiency of pump**

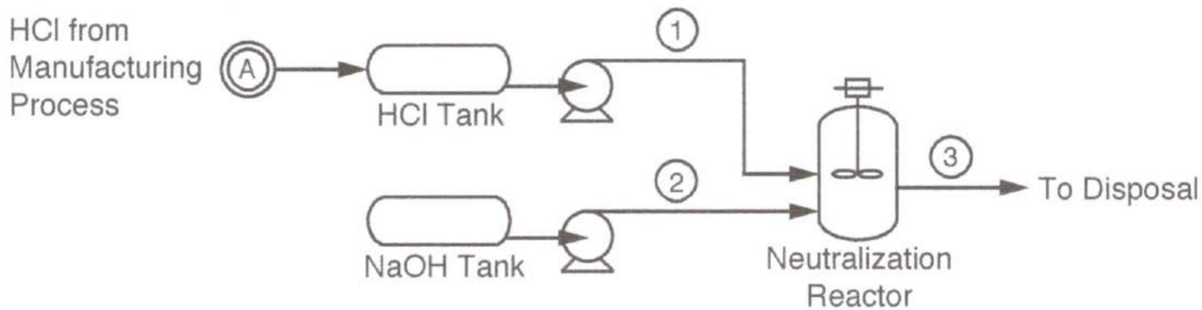
$$Pump\ Efficiency = \frac{Power\ delivered\ to\ the\ fluid}{Power\ to\ operate\ the\ pump} \qquad (7.12)$$

***Typical values for the efficiency of a centrifugal pump range from 70% to 90%.**

7.5 Pumps and turbines

7.5.1 Pumps

➢ **Process flow diagram for the acid neutralization process, including pumps**



Flows kg/h

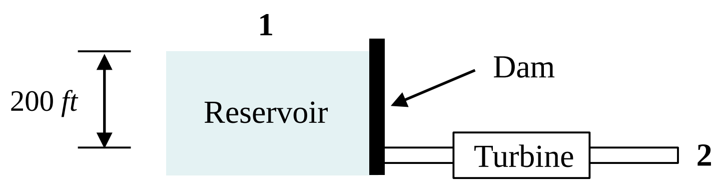
Line no. / Stream / Component	1 Acid feed	2 Base feed	3 Reactor outlet	ABC Chemical Co.	
HCl	6	—	—	Acid neutralization	
NaOH	—	6	—	1×10^8 L/yr	
H_2O	11594	6490	18096	Sheet no. 1	
Total	11600	6496	18096	Dwg by	Date
				Checked	1 Sep.2010

7.5 Pumps and turbines

7.5.2 Turbines

Example 7.10

A dam holds a reservoir of water that drives a turbine-powered generator to provide hydroelectric power. Water flows from a pipe in the bottom of the dam (220 *ft* below the top of the water) through the turbine at a rate of 1650 lb_m/s and then empties out of the turbine outlet pipe into the river below.



How much horsepower can the turbine theoretically produce? (For this problem, assume that the effects of friction and kinetic energy can be neglected.)

Rearrangement of mechanical energy equation for w_s:

$$\cancelto{0}{\frac{P_2-P_1}{\rho}}+\cancelto{0}{\frac{1}{2}\left(\alpha_2 v_{2,avg}^2-\alpha_1 v_{1,avg}^2\right)}+g\left(z_2-z_1\right)=w_s-\cancelto{0}{w_f} \quad \Rightarrow \quad w_s=g\left(z_2-z_1\right)=\left(\frac{32.2\ ft}{s^2}\right)\left(-200\ ft\right)\left(\frac{1\,s^2\ lb_f}{32.2\ lb_m\ ft}\right)\left(\frac{1\,hp\ s}{550\ ft\ lb_f}\right)=-0.364\,\frac{hp\ s}{lb_m}$$

so, $$w_{turbine}=-w_s=0.364\,\frac{hp\ s}{lb_m}$$

$$Power=\left(\frac{0.364\ hp\ s}{lb_m}\right)\left(\frac{1650\ lb_m}{s}\right)=600\ hp$$

7.5 Pumps and turbines

7.5.2 Turbines

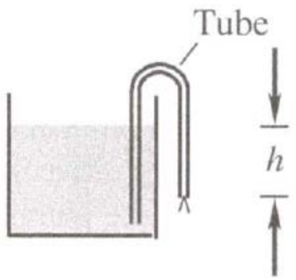
- **Efficiency of turbine**

$$\text{Turbine Efficiency} = \frac{\text{Power delivered by turbine}}{\text{Power extracted from the fluid}} \qquad (7.13)$$

Homework problems

Homework problem 5.

One way to drain a liquid out of a container that has high walls is to use a *siphon*: a tube is placed with one end in the liquid and the other end over the wall of the container as shown in below. The tube is then filled with the liquid (usually by applying suction to the open end). As long as the elevation of the open end is lower than that of the top of the liquid, the liquid will flow out of the tank.

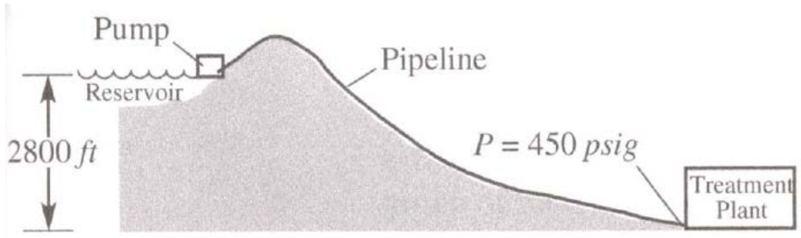


a. Derive an equation for the velocity coming out of the tube as a function of h. What happens when h approaches 0? (Assume that friction negligible and that the top of the water in the tank is essentially stationary.)
b. What volumetric flow rate will result if h = 7 *cm* and the tube diameter equals 1 *cm*?

Homework problems

Homework problem 10.

A system consisting of a pump and pipeline is being designed to deliver water (density = 62.4 lb_m/ft^3) from a reservoir in the mountains down to the city 2800 *ft* below. The water must arrive at a water treatment plant in the city at a pressure of 450 *psig*.



The flow rate of water is to be 63.5 *gal/s*, for which the friction in the pipeline is estimated to be 4.9 $hp\ s/lb_m$. How many horsepower must the pump deliver?

Hints:

1. Power is work/time and can be expressed in units of horsepower (*hp*) where 1 *hp* = 550 $ft\ lb_f/s$
2. Assume that the kinetic energy term for flow in the pipe (e.g., at the entrance to the treatment plant) is small compared with the other energy terms.
3. Remember that each term in the mechanical energy equation has units of energy (or work) per mass of fluid, while power has units of energy (or work) per time.