Introduction to Chemical Engineering

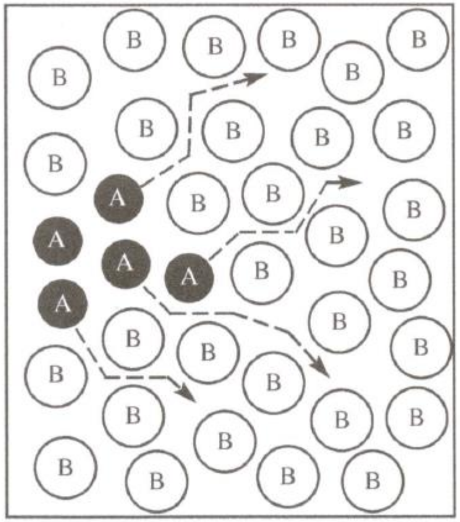
Chapter 08

Mass Transfer
(*Mixing the Acid and Base*)

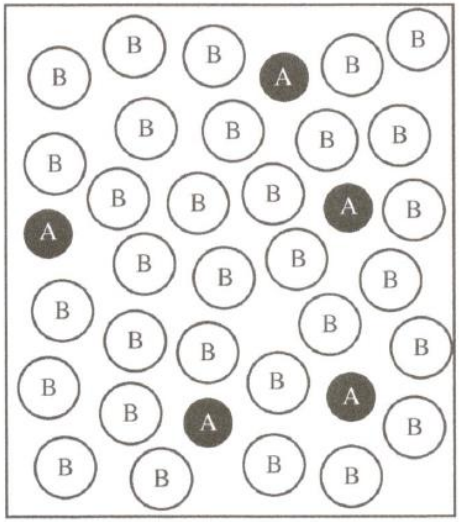
8.1 Molecular diffusion

- The thermal motion of all (liquid or gas) particles at temperature above absolute zero.
- A function of temperature, viscosity of the fluid and the size (mass) of the particles.

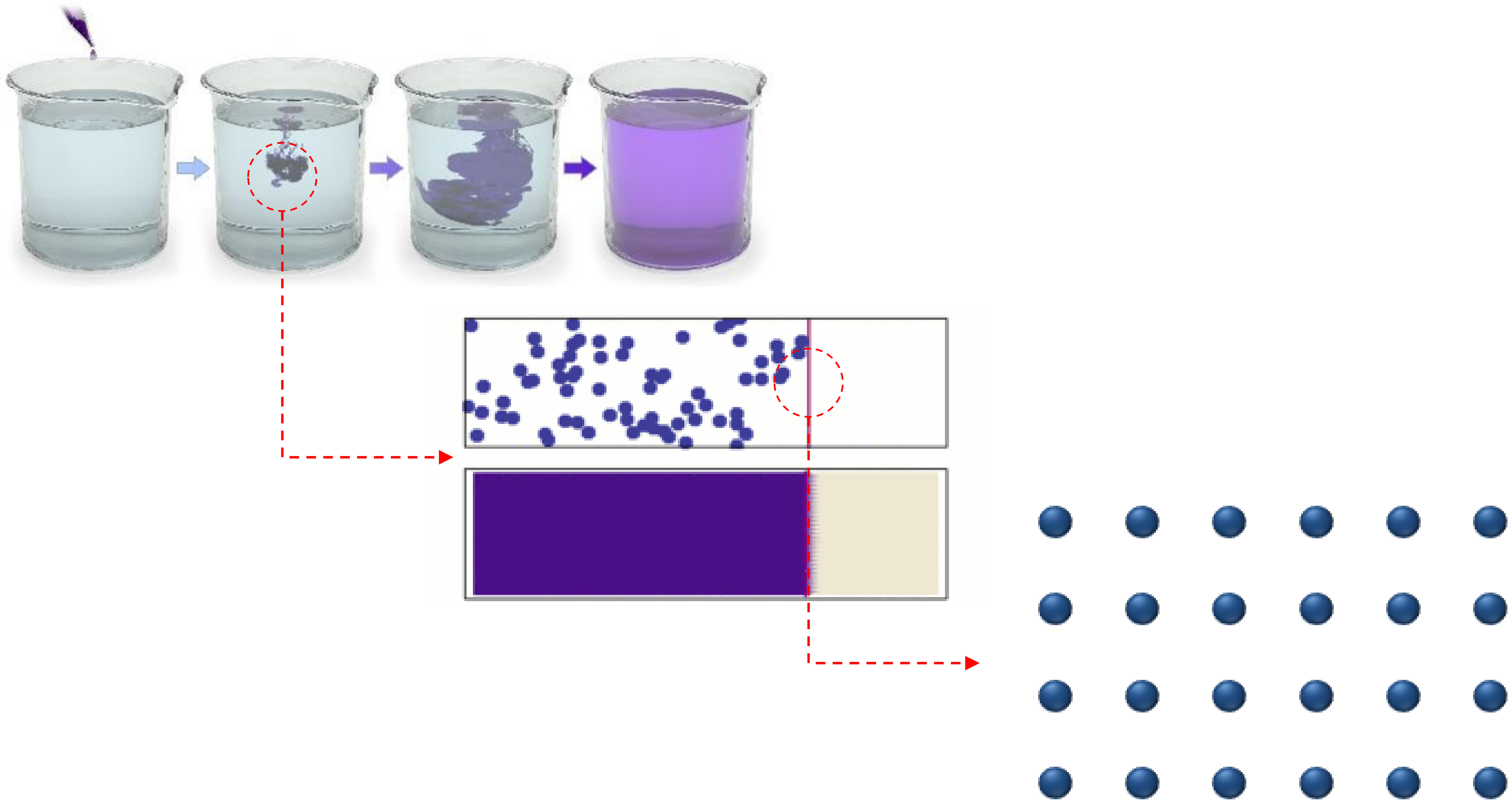
Molecular diffusion of A through B



New location of A

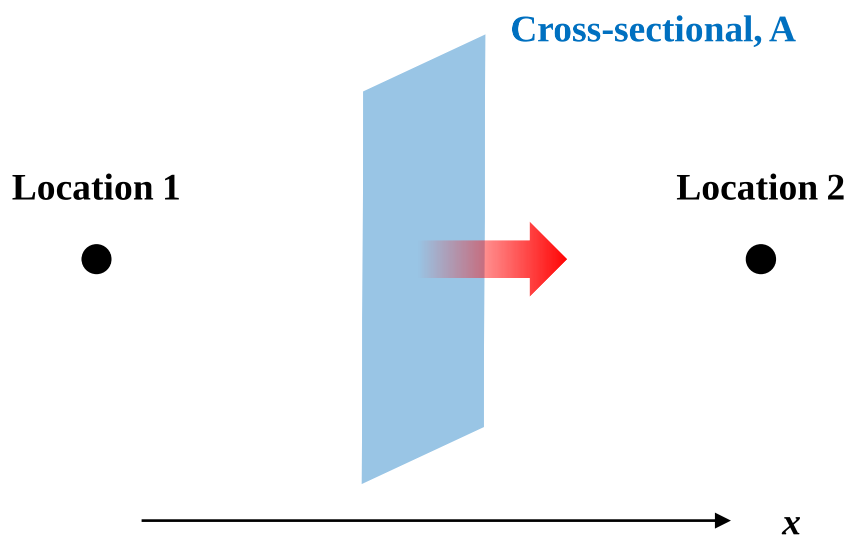


8.1 Molecular diffusion



8.1 Molecular diffusion

- **Fick's law**



$$\dot{N}_{A,x} = -D_{AB} A \frac{c_{A,2} - c_{A,1}}{x_2 - x_1} = -D_{AB} A \frac{\Delta c_A}{\Delta x}$$

$\dot{N}_{A,x} = diffusion\ transfer\ rate\ of\ species\ A\ (e.g.,\ gmol/s)$

$A = cross-sectional\ area\ across\ which\ diffusion\ occurs\ (e.g.,\ cm^2)$

$D_{AB} = the\ binary\ diffusivity\ of\ species\ A\ in\ species\ B\ (e.g.,\ cm^2/s)$

$c_A = the\ concentration\ of\ species\ A\ (e.g.,\ gmol/L)$

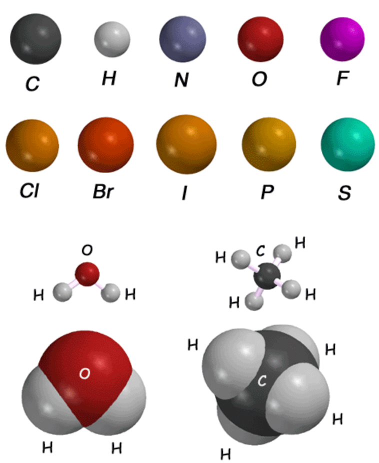
8.1 Molecular diffusion

- **Driving force and resistance**

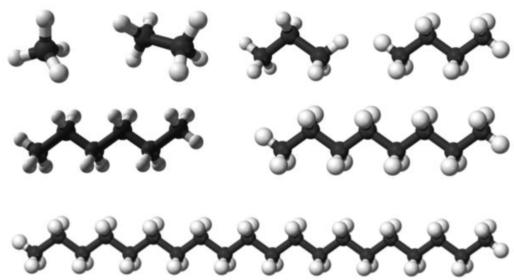
$$transfer\ rate = \frac{driving\ \ force}{resistance} = \frac{\Delta c_A}{\Delta x / D_{AB} A} \qquad \textbf{vs.} \qquad current = \frac{voltage}{resistance} = \frac{V}{R}$$

- **Diffusivity (diffusion coefficient)**

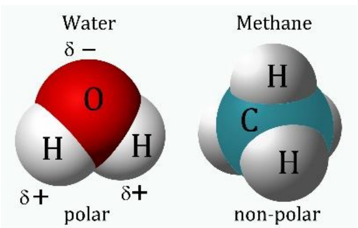
Molecular size

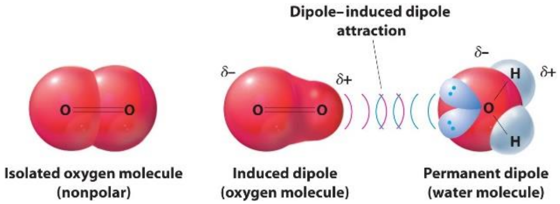


Molecular shape



Molecular charge

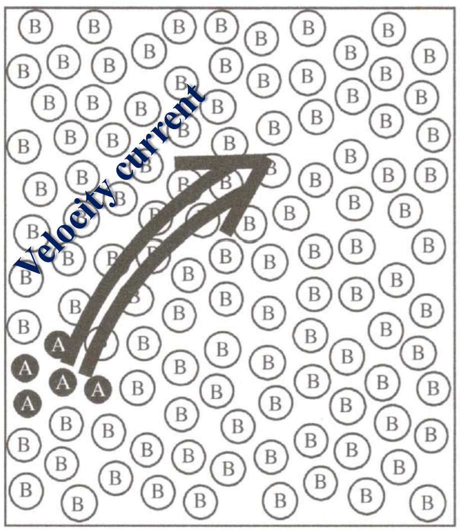




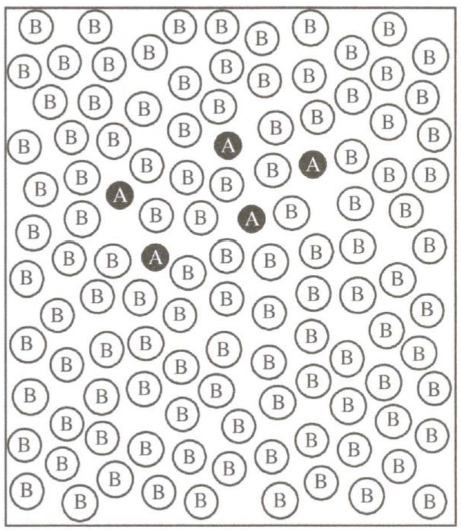
8.2 Mass convection

- The movement of groups of molecules within fluids such as liquids or gases.
- Convection takes place through advection, diffusion or both.

Mass convection of A and B



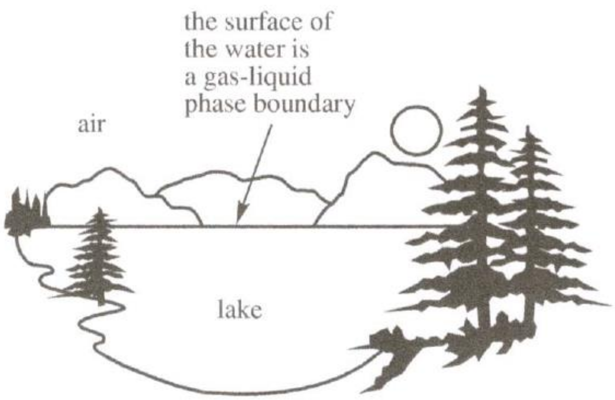
New location of A



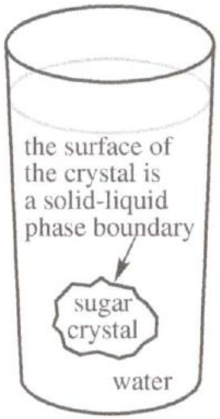
8.3 Mass convection with transfer across phase boundaries

- **Mass transfer at phase boundaries (interface)**

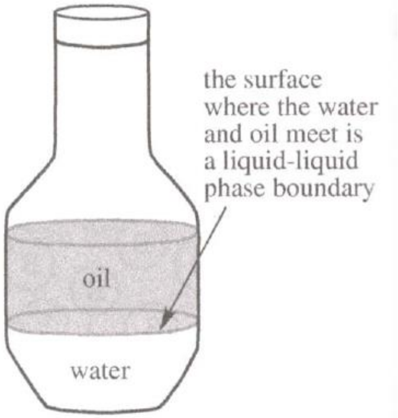
A gas-liquid boundary



A solid-liquid boundary

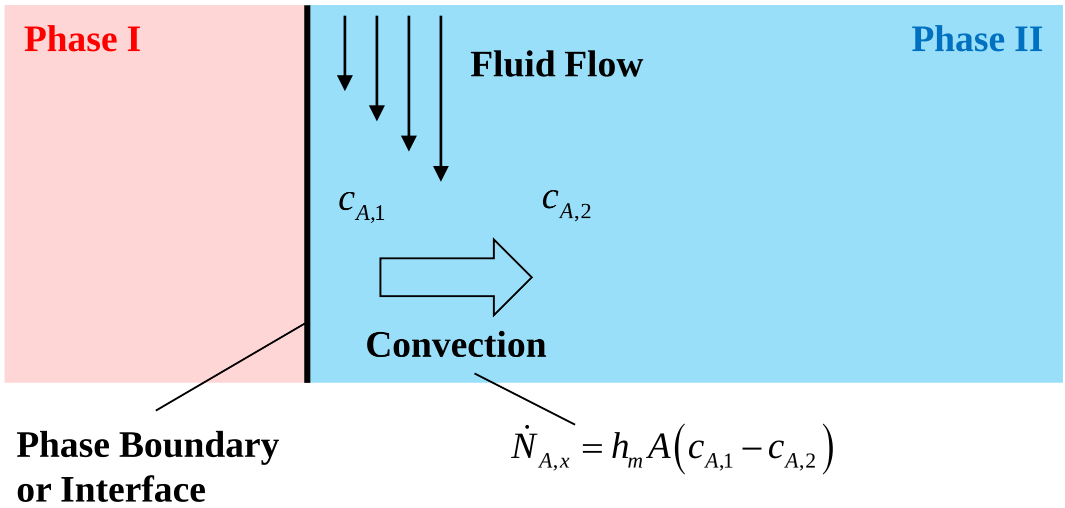


A liquid-liquid boundary



8.3 Mass convection with transfer across phase boundaries

➢ **Mass transfer at phase boundaries (interface)**



$$\dot{N}_{A,x} = h_m A\left(c_{A,1} - c_{A,2}\right)$$

$c_{A,1} = concentration\ of\ species\ A\ at\ the\ starting\ location$

$c_{A,2} = concentration\ of\ species\ A\ at\ the\ ending\ location$

$\dot{N}_{A,x} = convection\ transfer\ rate\ of\ species\ A$

$h_m = mass-transfer\ coefficient\ (diffusion\ and\ fluid\ motion,\ cm/s)$

$A = cross-sectional\ area\ through\ the\ transfer\ takes\ place$

8.3 Mass convection with transfer across phase boundaries

Example 8.1

A particular lake is a favorite site for recreation but is also windy, causing enough water evaporation that the level of the lake drops throughout the summer. It turns out that water skiing on the lake is safe until the water level drops 1 *m* from the early-spring level.

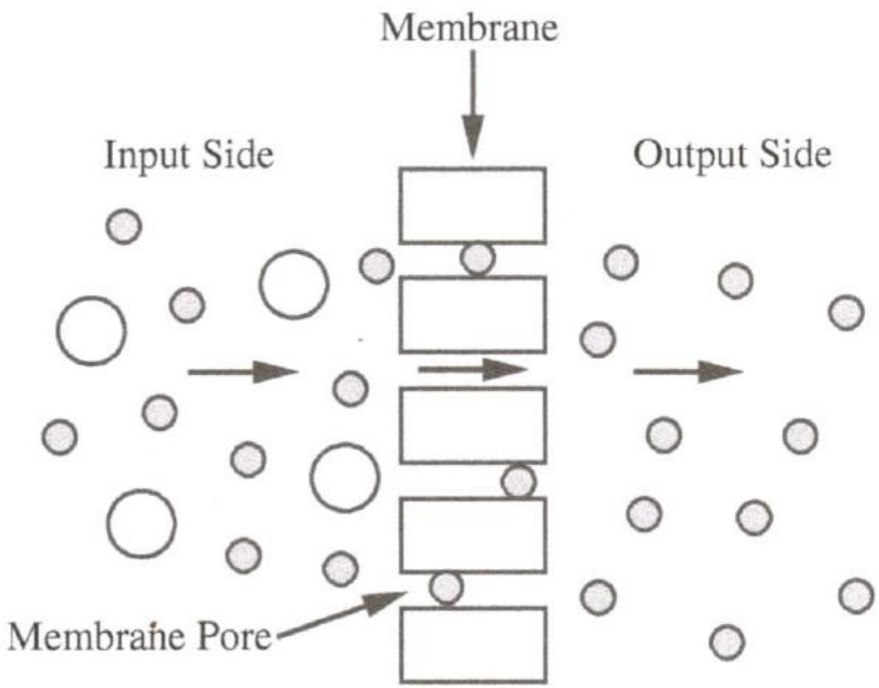
a. How much volume will the lake lose per day due to evaporation?
b. How long from early spring will it take for the water level to drop 1 *m*?

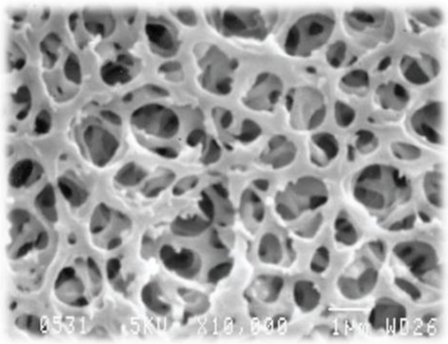
The following values pertain:

concentration of water in the air at the water surface	1.0×10^{-3} *kgmol/m*^{3}
concentration of water in the wind	0.4×10^{-3} *kgmol/m*^{3}
area of the lake	1.7 *mi*^{2}
mass-transfer coefficient of water from the lake surface	0.012 *m/s*
density of the lake water	1000 *kg/m*^{3}

8.4 Multi-step mass transfer

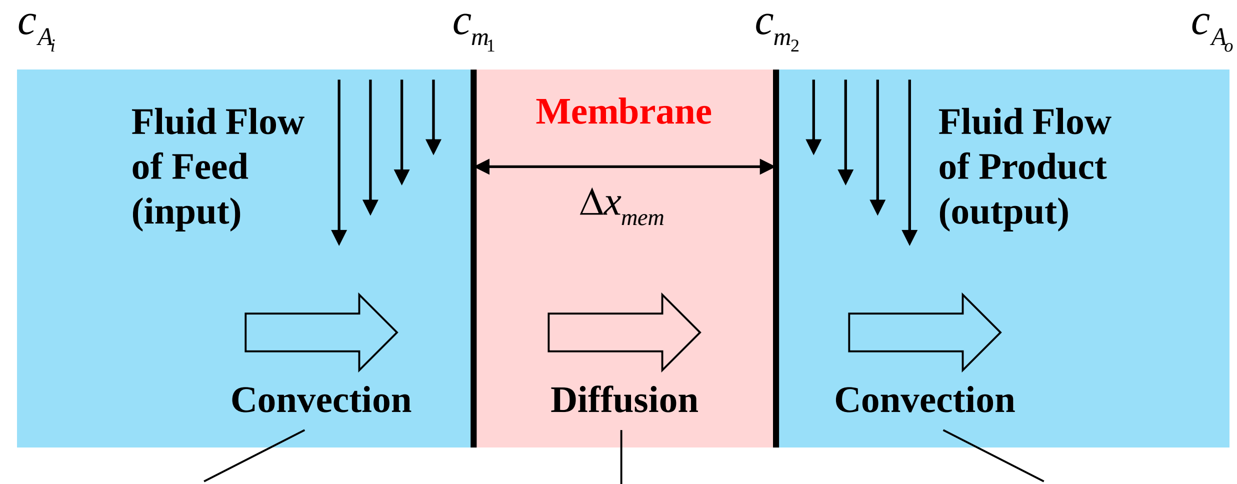
8.4.1 Membrane separation





8.4 Multi-step mass transfer

8.4.1 Membrane separation



$$\dot{N}_A = h_{m,i} A\left(c_{A_i} - c_{m_1}\right)$$

$$\dot{N}_A = D_{A,mem} A \varepsilon_{pore} \frac{c_{m_1} - c_{m_2}}{\Delta x_{mem}}$$

$$\dot{N}_A = h_{m,o} A\left(c_{m_2} - c_{A_o}\right)$$

8.4 Multi-step mass transfer

8.4.1 Membrane separation

$$\dot{N}_A = h_{m,i}A\left(c_{A_i} - c_{m_1}\right) \longrightarrow c_{A_i} - c_{m_1} = \frac{\dot{N}_A}{h_{m,i}A}$$

$$\dot{N}_A = D_{A,mem}A\varepsilon_{pore}\frac{c_{m_1} - c_{m_2}}{\Delta x_{mem}} \longrightarrow c_{m_1} - c_{m_2} = \frac{\dot{N}_A \Delta x_{mem}}{D_{A,mem}A\varepsilon_{pore}}$$

$$\dot{N}_A = h_{m,o}A\left(c_{m_2} - c_{A_o}\right) \longrightarrow c_{m_2} - c_{A_o} = \frac{\dot{N}_A}{h_{m,o}A}$$

$$c_{A_i} - c_{A_o} = \frac{\dot{N}_A}{h_{m,i}A} + \frac{\dot{N}_A \Delta x_{mem}}{D_{A,mem}A\varepsilon_{pore}} + \frac{\dot{N}_A}{h_{m,o}A} = \dot{N}_A\left(\frac{1}{h_{m,i}A} + \frac{\Delta x_{mem}}{D_{A,mem}A\varepsilon_{pore}} + \frac{1}{h_{m,o}A}\right)$$

$$\dot{N}_A = \frac{c_{A_i} - c_{A_o}}{\frac{1}{h_{m,i}A} + \frac{\Delta x_{mem}}{D_{A,mem}A\varepsilon_{pore}} + \frac{1}{h_{m,o}A}} = \frac{overall\ driving\ force}{\sum resistances}$$

Limiting resistance?

8.4 Multi-step mass transfer

Example 8.2

Liquid B flows on one side of a membrane, and liquid C flows along the other side of the membrane. Meanwhile, species A present in both liquids transfers from liquid B through the membrane and into liquid C. The following data pertain:

concentration of A in liquid B	5.0 *M*
concentration of A in liquid C	5.0 *M*
thickness of the membrane	200 μm
diffusivity of species A in the membrane	1.0 x 10^{-9} m^2/s
area of membrane	1 m^2
porosity of membrane	70%
mass-transfer coefficient on side B	7.0 x 10^{-4} m/s
mass-transfer coefficient on side C	3.0 x 10^{-4} m/s

a. What are the relevant mass-transfer mechanisms?
b. What is the transfer rate of A from the B side to the C side?
c. Calculate the limiting resistance.
d. Which of the following is most likely to increase the transfer rate?

    A. Increasing the flow rate of liquid B
    B. Decreasing the thickness of the membrane
    C. Decreasing the magnitude of the diffusivity
    D. Increasing the flow rate of liquid C

8.4 Multi-step mass transfer

Example 8.3

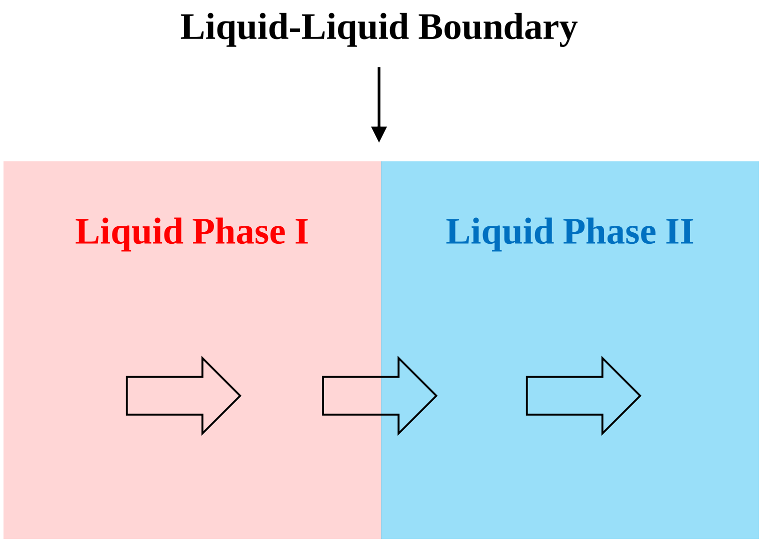
In patients with severe kidney disease, urea must be removed from the blood with a "hemodialyzer." In that device, the blood passes by special membranes through which urea can pass. A salt solution ("dialysate") flows on the other side of the membrane to collect the urea and to maintain the desired concentrations of vital salts in the blood. One geometry for hemodialyzer design is with flat membranes in a rectangular system. For such a geometry, consider the following typical values:

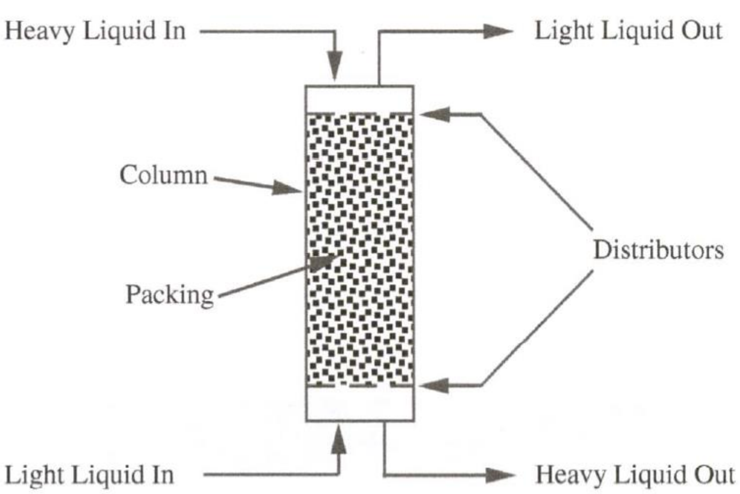
Blood side:	
mass-transfer coefficient for the urea	0.0019 *cm/s*
average urea concentration within the dialyzer	0.020 *gmol/L*
Dialysate side:	
mass-transfer coefficient for the urea	0.0011 *cm/s*
average urea concentration within the dialyzer	0.003 *gmol/L*
Membrane:	
thickness	0.0016 *cm*
diffusivity of urea in the membrane	1.8 x 10-5 cm^2/s
total membrane area	1.2 m^2
porosity	20%

a. Based on these values, what is the initial removal rate of urea? (Note: This rate will decrease as the urea concentration in the blood decreases.)
b. One might be tempted to try to increase the removal rate of urea by developing better hemodialyzer membranes. Based on analysis of these characteristics, is such an effort justified?

8.4 Multi-step mass transfer

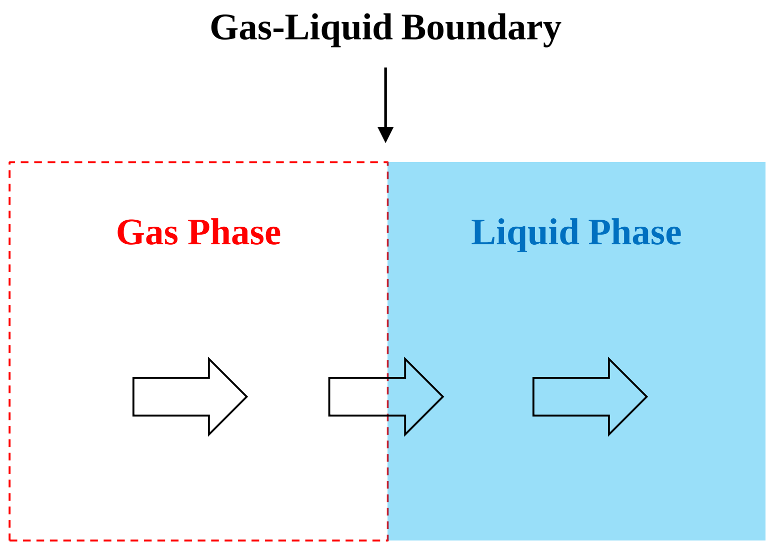
8.4.2 Liquid-liquid extraction





8.4 Multi-step mass transfer

8.4.3 Gas-liquid absorption



Homework problems

Homework problem 2.

Some discarded solid chemical waste dissolves slowly in a large drain pipe in which the water is stagnant. On a particular day, the dissolved chemical has a concentration of 0.16 *M* near the solid and is essentially 0 at a location 13.6 *m* further along the pipe. The transfer rate (moles per time) of the chemical through the water in the pipe between those two points is 7.3 *gmol/min*.

a. What equation describes this kinds of transfer?
b. Several days later, the chemical concentration near the solid has decreased to 0.105 *M* and is essentially 0 at a location 9.9 *m* away. What will be the transfer rate between the point near the solid and the point 9.9 *m* away on this later day?
c. On the next day, the heavy rains cause a current of water to flow through the drain pipe where the dissolving solid is located. The solid now dissolves twice as fast as on the previous day. If the concentrations are still 0.105 *M* near the solid and 0 at the more remote locations, and if the cross-sectional area for transfer is 0.3 m^2, what is the value of the mass-transfer coefficient at this time?

Homework problems

Homework problem 3.

The binary diffusivity of a particular salt in water at room temperature is 3.2 x 10^{-4} cm^2/s. That salt in solid form dissolves at the bottom of beaker of water (8 cm high), while water flows across the top of the beaker at a high velocity, as shown.

The concentration of the dissolved salt equals its saturation value (c_{sat}) at the top surface of the solid, and it equals 0 at the top of the beaker (because of the high flow rate of the water flowing across.) When the water inside the beaker is stirred at a particular rate, the solid dissolves at a rate 4 times greater than when the water in the beaker is not stirred (sits stationary). What is the value of the mass-transfer coefficient in the stirred condition (numerical value and units)?

Homework problems

Homework problem 5.

The intermediate product of a chemical process is a chemical paste that is saturated with water. The paste is rolled into a thin layer and placed to dry in a flat area 30 *m* long by 15 *m* wide. Drying is enhanced by blowing hot air over the paste, and the mass-transfer coefficient is 0.017 *m*/*s*. Diffusion of water through the paste is rapid so that the concentration of water vapor at the surface of the paste remains constant at 0.002 *gmol*/*L*. Water must be removed from the paste at a minimum rate of 9.5 *L*/*min* (liquid volume). (Note: MW_{water} = 18 and ρ_{water} = 1 g/cm^3)

a. What mechanism controls the transfer of water from the paste into the air?
b. What is the molar rate of water removal that corresponds to a volumetric removal rate of 9.5 *L*/*min*?
c. What is the maximum concentration of water vapor allowable in the air (far away from the paste surface) if water must be removed at a rate of at least 9.5 *L*/*min*?
d. Assuming that the mass-transfer coefficient remains constant, what can be done to increase the rate of water removal from the paste?